Thermal Patterns in Peripheral Regions of Breast during Different Stages of Development

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Abstract

Background: Mathematical modelling of bio thermal processes is widely used to enhance the quantitative understanding of thermoregulation system of human body organs. This quantitative knowledge of thermal information of various human body organs can be used for developing clinical applications. In the past the investigators have studied thermal distribution in hemispherical shaped human breast in presence of spherical shaped tumor. The shape and size of the breast as well as tumor may also affect thermal distribution which can have serious implications in thermography. In the present paper a model of thermal disturbances in peripheral regions of ellipsoidal shaped breast for two dimensional steady state case. The modelling study will provide biomedical scientist vital insights of thermal changes occurring due to shape and size of breast which can influence the development of protocols of thermography for diagnosis and treatment of tumors in women’s breast.

Method: We have incorporated the significant parameters like blood flow, metabolic activity and thermal conductivity in the thermal model for normal and malignant tissues. The controlled metabolic activity has been incorporated for normal tissues. The peripheral regions of breast are divided into three major layers mainly epidermis, dermis and sub dermal tissues. The outer surface of the breast is assumed to be exposed to the environment and the heat loss takes place by conduction, convection, radiation and evaporation. The finite element approach is employed to obtain the solution.

Results: By selecting appropriate model parameters we have shown the spatial thermal variation in different stages of breast according to their shapes which could be replicated by the proposed model.

Conclusions: The proposed model was successfully employed to study the impact of different sizes and shapes in peripheral regions of elliptical shaped woman’s breast. The proposed model is more realistic in terms of shape and size of woman’s breast in comparison to earlier models reported in the literature. The changes in slope of the thermal curves at the junctions of various peripheral layers are due to the non homogenous nature of the region.

Keywords: Thermal patterns; Stages of breast development; Semi elliptical shaped breast; Finite element method; Coaxial elliptical sectoral element

Introduction

The temperature changes within the human body in relation to disease have been recognized for many centuries. In particular the elevated body temperature has been used as an indicator of illness and often as an indicator of the progression of a disease. The medical and clinical practitioners use temperature as an indicator of tissue response under various clinical and health situations [1-3]. In order to understand the relation of tissue temperature with disease, it is of crucial interest to understand the thermoregulation of human body. The human body maintains its body core temperature at a uniform temperature i.e., 37°C by maintaining balance between metabolic heat generation in the body tissues and heat loss to the environment from the skin surface. The skin and subdermal tissues (peripheral) region is the medium for heat transport from body core to the body surface. This peripheral region is also known as peripheral region is a non homogeneous medium and consists of three layers namely epidermis, dermis and sub dermal tissues [4,5]. The dermis is made up of matted masses of connective tissues, elastic fibres, blood vessels, lymphatic and nerves. There are no blood vessels in the epidermis. The population density of blood vessels in the dermis is very thin near the interface of epidermis, but it increases gradually and becomes almost uniform in the sub dermal part [6].

Apart from the biophysical parameters like thermal conductivity, blood flow, and metabolic activity, the structure, shape and size of the body organs also influence this tissue temperature of the human or animal body. The shape, size and structure of the breast vary depending on the stages of development, age, community etc. [7]. The female breast is made up of glandular, fatty, and fibrous tissues. There is a layer of fatty tissue that surrounds the breast glands and runs throughout the entire breast. This fat gives breasts their soft consistency, size and shape. Inside the glandular tissues are the functional parts of the breast.

The development of breast takes place in five stages as shown in Figure 1. In the first stage only the tip of the nipple is the raised part and otherwise it is flat. It is the childhood stage i.e., less than 7 to 8 years, before puberty begins and the breasts have not started to
develop. In the second stage of development the buds appear, breast and nipple are raised, and the areola (dark area of skin that surrounds the nipple) enlarges. This stage occurs during the age of 8 to 13 years. In the third stage breasts are slightly larger with glandular breast tissue present in them. Girls usually reach this stage when they are 12 to 14 years old. The areola and nipple become raised and form a second mound above the rest of the breast in the fourth stage of development [8]. Girls often reach this stage when they are 12 to 15 years old and many girls reach stage 4 at ages that are outside this range. Finally in the fifth stage mature adult breast is developed during the age of 15 to 18 years and in this stage the breast becomes rounded and only the nipple is raised.

From the literature survey it is observed that no attempt is reported till date to study thermal patterns in elliptical shaped human breast. Further no research workers have taken into account the size and shape of the breast depending on the various stages of development of the breast. In the present study a finite element model has been developed to study thermal patterns in peripheral region of semi elliptical shaped human breast under different stages of development for a two dimensional steady state case. The effect of shape and size of breast on thermal patterns in the peripheral region of the breast is analysed.

Mathematical Model

The partial differential equation for heat flow in the peripheral region which was given by [33]:

\[(\rho c) \frac{\partial T}{\partial t} = \nabla \cdot (K \nabla T) + m_b \frac{\partial}{\partial r} \left( T - T_b \right) + S\]

(1)

Where the effect of blood flow and metabolic heat generation is given by the term \(m_b \frac{\partial}{\partial r} \left( T - T_b \right)\) and \(S\) respectively. Here \(K=\)Thermal conductivity of tissue, \(m_b=\)Blood mass flow rate, \(c_p=\)Specific heat of blood, \(T_b=\)Arterial blood temperature, \(T=\)Tissue temperature at position \(r\) measured perpendicularly from the skin surface, \(c=\) specific heat of the tissues at time \(t\), \(S=\)Rate of metabolic heat generation, \(\rho=\)Tissue density. We consider \(T_A= T_b=\)Body core temperature, as the blood flows in arteries from the body core at body core temperature.

The human breast is assumed to be ellipsoidal in shape with upper semi ellipsoidal region projecting out from the trunk of the body and lower semi ellipsoidal portion is considered to be a part of the body core. The deep tissues of breast consisting of muscles, glands and fat are considered to be a part of the breast core. Above the breast core is the peripheral region.

The equation (1) for thermal patterns in living tissues for a two dimensional steady state case in elliptical coordinates is given by:

\[\frac{1}{\sinh^2 \mu + \sin^2 \nu} \left[ K \frac{\partial}{\partial \mu} \left( \frac{\partial T}{\partial \mu} \right) + K \frac{\partial}{\partial \nu} \left( \frac{\partial T}{\partial \nu} \right) \right] + m_b \frac{\partial}{\partial r} \left( T - T_b \right) + S = 0\]

(2)

Here \(d\) is the eccentricity of elliptical layer, \(\mu\) and \(\nu\) are the radial and angular coordinates for the ellipsoidal shaped human breast.

The outer surface of the region is exposed to the environment and heat loss at this surface takes place mainly due to convection, radiation, and evaporation [9,10]. Hence the boundary condition imposed at the outer surface is given by:

\[-k \frac{\partial T}{\partial r} = h(T - T_\text{a}) + LE \text{ at } \mu = \mu_3 , \nu \in (0, \pi)\]

(3)

Where, \(h=\)heat transfer coefficient, \(T_a=\)atmospheric temperature, \(L=\)latent heat of evaporation, \(E=\)Rate of evaporation & represents flux normal to the skin surface.

The condition at the inner boundary is imposed for two cases as given below.

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**Figure 1**: Different stages of development of breast [8].
Case I boundary condition

At medium and higher atmospheric temperatures the inner boundary is maintained at uniform body core temperature $T_b$. Thus inner boundary is assumed to be at constant temperature $T_b$. Hence the condition at inner boundary is given by:

$$T(\mu_b, \nu) = T_b$$

(4)

Where $\mu_b$ is the radial position of inner boundary.

Case II boundary condition

At low atmospheric temperatures, the core temperature of the human breast is variable along angular direction $\nu$. This is because the warm blood flows in arteries at 37°C from the core of the trunk to the core of the human breast and the same blood reaching extremities cools down and returns back from extremities of the breast through veins at lower temperature than the body core temperature. Hence the following boundary condition is imposed.

$$T(\mu_{bc}, \nu) = F(\nu)$$

(5)

$$F(\nu) = a_1 + a_2\nu + a_3\nu^2$$

(6)

Here $a_1$, $a_2$ and $a_3$ are constants.

The value of $a_1$, $a_2$, $a_3$ are found by using the conditions

$$T(\mu_{bc}, \nu) = \alpha \text{ at } \nu = 0$$

(7)

$$T(\mu_{bc}, \nu) = \beta \text{ at } \nu = \pi/2$$

(8)

$$T(\mu_{bc}, \nu) = \gamma \text{ at } \nu = \pi$$

(9)

Hence the value of $\alpha$, $\beta$ and $\gamma$ are constants, which can be assigned the values based on the temperature at selected points of the core of human breast.

The peripheral region of the breast is divided into the three layers namely epidermis, dermis and subcutaneous tissues, which are considered to be elliptic with eccentricity $d_1$, $d_2$ and $d_3$ respectively. The coaxial elliptical sectoral elements have been employed to discretize the region as given in Figure 2. The whole region is divided into 18 elements. This division of each layer into different number of elements of different sizes has been done in order to match with the geometry and physiological properties of the region [33].

The equation (2) along with the boundary conditions (3) and (4) is written in the variational form [20] as given below:

$$J^{(e)} = \frac{1}{2} \int \int K^{(e)} \left( \frac{\partial T^{(e)}}{\partial \mu} \right)^2 + \left( \frac{\partial T^{(e)}}{\partial \nu} \right)^2 \right) d\mu d\nu$$

$$+ A^{(e)} \left( m b_h \left( T_{bc}^{(e)} - T \right)^2 - 2 S^{(e)} T^{(e)} \right) d\mu d\nu$$

$$+ \frac{1}{2} \int \int A^{(e)} \left[ h \left( T^{(e)} - T_a \right)^2 + 2 L E^{(e)} T^{(e)} \right] d\mu d\nu$$

for $e = 1(1)18$ (10)

Here $A^{(e)} = d^{(e)}(\sinh 2f^{(e)} + \sin 2f^{(e)})$, $\mu_i$ and $\mu_j$ are boundaries of the eth element, $K^{(e)}$, $M^{(e)}$, $S^{(e)}$, $T_{bc}^{(e)}$ and $T^{(e)}$ denote the values of $K$, $M$, $S$, $T_a$ and $T$ respectively in the eth layer [25]. $\lambda(e) = 1$ for elements along the surface and $\lambda(e) = 0$ for all the elements which are not along the outer surface.

The following bilinear shape function for variation of temperature within each element has been taken as [6].

$$T^{(e)} = c_1^{(e)} + c_2^{(e)} \mu + c_3^{(e)} \nu + c_4^{(e)} \mu \nu$$

(11)

Where $c_1^{(e)}$, $c_2^{(e)}$, $c_3^{(e)}$ and $c_4^{(e)}$ are constants for the eth element.

Using above values of parameters the integral (10) are evaluated and assembled as follows:

$$I = \sum_{i=1}^{n} I^{(e)}$$

(12)

This leads to a system of linear algebraic equations given below:

$$[X]_{28 \times 28} \left[ \begin{array}{c} T_{bc}^{(e)} \end{array} \right] = \left[ Y \right]_{28 \times 1}$$

(13)

Here, $T = \left[ T_1 \ T_2 \ T_3 \right]$ and $T_{bc}^{(e)}$ denote the matrix of order $28 \times 28$ and $Y$ is system vector of order $28 \times 1$. The two types of boundary conditions (4) and (5) are incorporated at the inner core of the breast in these systems of equations (13). The Gauss elimination method has been used to obtain the solution of (13). A computer program in MATLAB is developed to find numerical solution to the entire problem. The time taken for simulation is nearly 2 minutes on Core(TM) i3 CPU M 330 @ 2.13 GHz processing speed and 3 GB memory.

Numerical Result and Discussion

Numerical result

The numerical results are obtained by using the values of physical and physiological constants [11,12] as given in Table 1.

α, β and γ boundary condition the following values are assigned to the constants

\[
\begin{align*}
\alpha & = 37^\circ C, \\
\beta & = 36^\circ C, \\
\gamma & = 37^\circ C
\end{align*}
\]

The numerical results have been computed and graphs are plotted for the different cases of atmospheric temperatures and different values of rate of evaporation [9-11].

### Discussion

Graphs have been plotted for thermal patterns along radial and angular direction which are shown in Figures 3 to 26. The Figures 3 to 18 are for the case I boundary condition and Figures 19 to 26 are for case II boundary condition.

#### Case I boundary condition

Figures 3 to 6 show thermal patterns for \( T_a=23^\circ C, \ E=0.48 \times 10^{-3} \) gm/cm\(^2\)min case I, eccentricity of breast core \( d_1=0.0030 \) for the stages II, III, IV and V of breast development respectively. In these Figures it is observed that the slope of the curve changes at the junctions of the different layers in the region.

In Figure 6 for stage V the fall in temperature is more as compared to Figure 3 for stage II.

Figures 7 to 10 show thermal patterns for \( T_a=33^\circ C, \ E=0.48 \times 10^{-3} \) gm/cm\(^2\)min case I, eccentricity of breast core \( d_1=0.0030 \) and for the stages II, III, IV and V of breast development respectively.

### Table 1: Values of parameters [16].

<table>
<thead>
<tr>
<th>Atmospheric temperature ( T_a (^\circ C) )</th>
<th>( M_{\text{max}}=m(bcb)_{\text{max}} ) Cal/cm(^2)-min deg,C</th>
<th>( S_{\text{max}}=m ) cal/cm(^2) min</th>
<th>( E=gm/cm^2/min )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15°C</td>
<td>0.003</td>
<td>0.0357</td>
<td>0</td>
</tr>
<tr>
<td>23°C</td>
<td>0.018</td>
<td>0.018</td>
<td>0.0, 0.24 \times 10^{-3}, 0.48 \times 10^{-3}</td>
</tr>
<tr>
<td>33°C</td>
<td>0.315</td>
<td>0.018</td>
<td>0.0, 0.24 \times 10^{-3}, 0.48 \times 10^{-3}, 0.72 \times 10^{-3}</td>
</tr>
</tbody>
</table>

Case I boundary condition

The numerical results have been computed for following four sample layers of peripheral region of breast based on stage II to V respectively as given below:

- Stage -I \( T_a=23, \ E=0.00048 \) gm/cm\(^2\)min
- Stage - II, \( T_a=23, \ E=0.00048 \) gm/cm\(^2\)min
- Stage - III, \( T_a=23, \ E=0.48 \times 10^{-3} \) gm/cm\(^2\)min
- Stage - IV, \( T_a=23, \ E=0.48 \times 10^{-3} \) gm/cm\(^2\)min
- Stage - V, \( T_a=23, \ E=0.48 \times 10^{-3} \) gm/cm\(^2\)min

For any ellipse we can define a number of related eccentricities depending upon sample of a human organ under study. The following sets of eccentricities have been taken as a particular case:

- Set: 1 \( d_1=0.0030 \) cm, \( d_2=0.0028 \) cm, \( d_3=0.0025 \) cm.
- Set: 2 \( d_1=0.0030 \) cm, \( d_2=0.0028 \) cm, \( d_3=0.0025 \) cm.
- Set: 3 \( d_1=0.0030 \) cm, \( d_2=0.0028 \) cm, \( d_3=0.0025 \) cm.
- Set: 4 \( d_1=0.0030 \) cm, \( d_2=0.0028 \) cm, \( d_3=0.0025 \) cm.

Here \( d_1 \) represents the eccentricity of the breast core. The eccentricities \( d_2 \) and \( d_3 \) are computed values based on \( d_1 \). For case II boundary condition the following values are assigned to the constants \( \alpha, \beta \) and \( \gamma \):

\[
\alpha = 37^\circ C, \beta = 36^\circ C, \gamma = 37^\circ C
\]

The expression for nodal information is given below:

Radial coordinates: \( \mu_i=r_i \) for \( i=k+4j \) and \( k=1 (1) 4 \)

The numerical results have been computed and graphs are plotted for the different cases of atmospheric temperatures and different values of rate of evaporation [9-11].
In these Figures it is also observed that the slope of the curve changes at junction of the different layers in the region. The temperature falls down as we move from breast core to the outer surface along radial direction. In Figure 10 for stage V it is observed
that the fall in temperature is more as compared to that in Figure 7 for stage II. This may be because the surface area of breast in stage V which is exposed to the environment is more as compared to that in stage II.

Figures 11 to 14 show the thermal patterns for Ta=23°C, E=0.48 × 10^{-3} gm/cm^2 min, case I and stage V for different eccentricities d_1=0, 0.25, 0.5 and 0.75 respectively. In Figure 14 for d_1=0.75 it is observed that the fall in the temperature is less as compared to Figure 11 for d_1=0.

Figures 15 to 18 show the thermal patterns for Ta=33°C, E=0.48 × 10^{-3} gm/cm^2 min, case I and stage V for different eccentricity d_1=0, 0.25, 0.5 and 0.75 respectively. In Figure 18 for d_1=0.75 it is observed that the fall in the temperature is less as compared to Figure 15 for d=0. This may be due to higher surface area of breast exposed to the environment for lower eccentricity d_1=0. Comparing Figures 3 to 10 with Figures 11 to 18, we observe that the fall in temperature is more at lower atmospheric temperature (i.e., T_a=23°C) than that for higher atmospheric temperature (i.e., T_a=33°C).
It is not possible to present the graphs for all the different values of parameters involved. Therefore the results for different eccentricities based on stages of breast development, atmospheric temperature and for different rate of evaporations are presented in the Tables 2 to 7. We observe in Tables 2 to 7 that surface area of the breast exposed to environment increases with the stages of development from stage II to V.

The Table 2 shows the nodal temperatures for \( T_a=15^\circ C \), E=0, same eccentricity \( d_1=0.0030 \) and different stages of development of breast. The nodal temperature \( T_2 \), \( T_3 \) and \( T_4 \) are higher for stage II and go on decreasing along with the stages III, IV and V. The maximum fall in nodal temperature is observed at the surface and the difference in \( T_4 \) between stages II and stage V is 0.30°C. The Table 3 shows the nodal temperatures for \( T_a=23^\circ C \), E=0.24 \( \times 10^{-3} \) gm/cm²/min, E=0.48 \( \times 10^{-3} \) gm/cm²/min, same eccentricity \( d_1=0.0030 \) and different stages of development of breast. In Table 3 we observe that the nodal temperature \( T_2 \), \( T_3 \) and \( T_4 \) decrease with the stages as we move from stage II to stage V. The difference in surface temperature for \( T_a=23^\circ C \), E=0.24 \( \times 10^{-3} \) gm/cm²/min between stage II and stage V is 0.64°C. Again for \( T_a=23^\circ C \), E=0.48 \( \times 10^{-3} \) gm/cm²/min the difference in surface temperature between stage II and stage V is 0.7518°C. In Table 3 we also observe that the difference in nodal temperatures between the stages increases with the increase in rate of evaporation.

<table>
<thead>
<tr>
<th>Temperatur e</th>
<th>Stage-II</th>
<th>Stage-III</th>
<th>Stage-IV</th>
<th>Stage-V</th>
<th>Temperature difference between stages II &amp; V</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>36.39</td>
<td>36.31</td>
<td>36.28</td>
<td>36.2</td>
<td>0.21</td>
</tr>
<tr>
<td>T3</td>
<td>35.512</td>
<td>35.45</td>
<td>35.4</td>
<td>35.305</td>
<td>0.307</td>
</tr>
<tr>
<td>T4</td>
<td>34.154</td>
<td>34.1509</td>
<td>34.0625</td>
<td>34.0625</td>
<td>0.4091</td>
</tr>
</tbody>
</table>

Table 2: Nodal temperatures for \( T_a=15 \), E=0 Case-I and different stages of development of breast.

The Table 4 shows the nodal temperatures for \( T_a=33^\circ C \), different values of rate of evaporation, same eccentricity \( d_1=0.0030 \), case I and different stages of development of breast. We observe that the nodal temperatures decrease with the stages, as we go from stage II to stage V. Also the nodal temperature in the different stages decrease with the increase in rate of evaporation. This is due to the fact that the surface area of breast increases with the stages of development and the surface area have the most significant effect on thermal patterns in the breast at the higher rate of evaporation because more heat loss takes place from outer surface to the environment due to higher rate of evaporation.

The Tables 5, 6 and 7 show the nodal temperatures due to different eccentricities and full matured stage (i.e., stage V) of the breast. The Table 5 shows the nodal temperatures for \( T_a=15^\circ C \), E=0, different eccentricities \( d_1=0.0, 0.25, 0.50, 0.75 \), stage V of development of breast and case I. The nodal temperature \( T_2 \), \( T_3 \) and \( T_4 \) are higher for \( d_1=0.75 \) and go on decreasing with decrease in eccentricity and \( (d_1=0.5, 0.25 \) and 0.0) the maximum fall in nodal temperature is observed at the surface and the difference in \( T_4 \) between \( d_1=0 \) and \( d_1=0.75 \) is 0.455°C. The Table 6 shows the nodal temperatures for \( T_a=23^\circ C \), E=0.24 \( \times 10^{-3} \) gm/cm²/min, E=0.48 \( \times 10^{-3} \) gm/cm²/min, different eccentricities \( d_1=0.0, 0.25, 0.50, 0.75 \), stage V of development of breast and case I. In Table 6 we observe that the nodal temperature \( T_2 \), \( T_3 \) and \( T_4 \) decrease with the eccentricities as we move from \( d_1=0.75 \) to \( d_1=0 \). The difference in surface temperature for \( E=0.24 \times 10^{-3} \) gm/cm²/min between \( d_1=0.75 \) and \( d_1=0 \) is 0.75°C. Again for \( T_a=23^\circ C \),
E=0.48 \times 10^{-3} \text{ gm/cm}^2\text{min}, the difference in surface temperature between d_1=0.75 to d_1=0 is 0.84°C. The Table 6 we observe that the difference in nodal temperatures due to different eccentricities increases with the increase in rate of evaporation.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Stage-II</th>
<th>Stage-III</th>
<th>Stage-IV</th>
<th>Stage-V</th>
<th>Temperature difference between stages II &amp; V</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>35.61</td>
<td>35.650</td>
<td>35.45</td>
<td>35.2</td>
<td>0.4105</td>
</tr>
<tr>
<td>T3</td>
<td>34.65</td>
<td>34.8</td>
<td>34.38</td>
<td>34.15</td>
<td>0.5091</td>
</tr>
<tr>
<td>T4</td>
<td>33.82</td>
<td>33.5</td>
<td>33.25</td>
<td>33.18</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Table 3: Nodal temperatures for Ta=23, Case-I and different stages of development of breast.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Stage-II</th>
<th>Stage-III</th>
<th>Stage-IV</th>
<th>Stage-V</th>
<th>Temperature difference between stages II &amp; V</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>35.22</td>
<td>35.12</td>
<td>35</td>
<td>34.75</td>
<td>0.476</td>
</tr>
<tr>
<td>T3</td>
<td>33.52</td>
<td>33.3</td>
<td>33.15</td>
<td>32.95</td>
<td>0.575</td>
</tr>
<tr>
<td>T4</td>
<td>32.25</td>
<td>32.1</td>
<td>31.8</td>
<td>31.5</td>
<td>0.7518</td>
</tr>
</tbody>
</table>

Table 4: Nodal temperatures for Ta=33, Case –I and different stages of development of breast.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Stage-II</th>
<th>Stage-III</th>
<th>Stage-IV</th>
<th>Stage-V</th>
<th>Temperature difference between stages II &amp; V</th>
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<tr>
<td>T1</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>35.05</td>
<td>35.25</td>
<td>35.38</td>
<td>35.45</td>
<td>0.398</td>
</tr>
<tr>
<td>T3</td>
<td>34.05</td>
<td>34.15</td>
<td>34.28</td>
<td>34.58</td>
<td>0.535</td>
</tr>
<tr>
<td>T4</td>
<td>31.75</td>
<td>32.98</td>
<td>33.05</td>
<td>32.4</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Table 5: Nodal temperatures for Ta=15, E=0, Case–I, Stage V and different eccentricities.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Stage-II</th>
<th>Stage-III</th>
<th>Stage-IV</th>
<th>Stage-V</th>
<th>Temperature difference between stages II &amp; V</th>
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</thead>
<tbody>
<tr>
<td>T1</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>37</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>34.98</td>
<td>35</td>
<td>35.2</td>
<td>35.35</td>
<td>0.37</td>
</tr>
<tr>
<td>T3</td>
<td>33.65</td>
<td>33.5</td>
<td>33.75</td>
<td>34.35</td>
<td>0.7</td>
</tr>
<tr>
<td>T4</td>
<td>31.35</td>
<td>31.6</td>
<td>31.98</td>
<td>32.19</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Table 6: Nodal temperatures for Ta=23, Case-I, Stage V and different eccentricities.

The Table 7 shows the nodal temperatures for Ta=33°C, different values of rate of evaporation, different eccentricities d_1=0.0, 0.25, 0.50, 0.75, stage V of development of breast and case I boundary condition. We observe that the nodal temperatures decrease with the decrease in the eccentricities, as we go from d_1=0.75 to 0.0. Also the nodal temperature decreases with the increase in rate of evaporation. Further the difference in nodal temperatures due to different eccentricities increases with the increase in rate of evaporation.

This is due to the fact that the surface area of breast increases with the decrease in eccentricity and the surface area has the significant effect on thermal patterns in the breast at the higher rate of evaporation because as more heat loss takes place from the outer surface to the environment due to higher rate of evaporation.
Table 7: Nodal temperatures for Ta=33, Case-I, Stage V and different eccentricities.

Case II boundary condition

Figures 19 to 22 show the radial and angular direction distribution in peripheral region of breast for case-II, eccentricity $d_1=0.0030$, Ta=15°C, E=0 and stage-II, III, IV and V of breast development respectively. In these Figures the effect of parabolic variation of inner core temperature of breast is clearly visible on temperature profiles in peripheral region of breast. Also the slope of the curve changes at the different radial positions at the interfaces of layers of peripheral region. The fall in temperature along radial direction is more in Figure 22 as compared to that Figure 19. Thus due to higher surface area exposed to environment in the stage-V as compared to that in the stage-II.

The fall in temperature is more in Figure 23 as compared to that in Figure 26. Thus the fall in temperature decreases with increase in eccentricities. This is because the surface area decreases with increase in eccentricities.

Figures 23 to 26, show the radial and angular thermal patterns in peripheral region of breast for case-II, stage-V, $d_1=0.0030$ and Ta=15°C, E=0.
Figure 23: Radial and angular thermal patterns in peripheral regions of breast for case-II, Stage-V, $d_1=0.0$ and $T_a=15^\circ C$, $E=0$.

Figure 24: Radial and angular thermal patterns in peripheral regions of breast for case-II, Stage-V, $d_1=0.25$ and $T_a=15^\circ C$, $E=0$.

Figure 25: Radial and angular thermal patterns in peripheral regions of breast for case –II, Stage-V, $d_1=0.50$ and $T_a=15^\circ C$, $E=0$.

Figure 26: Radial and angular thermal patterns in peripheral regions of breast for case –II, Stage-V, $d_1=0.75$ and $T_a=15^\circ C$, $E=0$.

The Table 8 shows the values of nodal temperatures for $T_a=15^\circ C$, $E=0$, case-II, $d_1=0.0030$ and different stages of development of breast. We observe that the difference in nodal temperatures in the stages II to stage V increases as we move from breast core to breast surface. The nodal temperatures decrease as we move from stage II to stage V.

Table 9 shows nodal temperatures for $T_a=15^\circ C$, $E=0$, case-II, stage V and different eccentricities of breast.

Table 8: Nodal temperatures for $T_a=15^\circ C$, $E=0$ Case-II and different stages of development of breast.
In Table 9 we observe that the difference in nodal temperatures for eccentricities 0 and 0.75 increase as we move from breast core towards the surface of the breast. Also the nodal temperature increases with the increase in eccentricities. This is due to decrease in surface area of breast with increase in eccentricity thereby reducing the heat loss from surface to the environment due to conduction, convection radiation and evaporation.

The present model can be used to obtain the similar results for different cases of atmospheric temperatures. The model is valid for higher atmospheric temperature up to 45°C and lower atmospheric temperature up to 5°C. But for freezing temperature below 5°C model has to be modified for cold injuries. For higher atmospheric temperature above 45°C the present model has to be modified for heat/burn injuries. The model can further be modified and extended for the problem involving abnormalities like cancer to generate the temperature profiles for patients suffering from cancer. The temperature profile for abnormal case (cancer) can be compared with those for normal case to identify the location of differences of temperature profiles which in turn can be used to detect the location of abnormalities like cancer. Further the models can be simulated for different size and type of tumors to generate the temperature profiles. This thermal information can again be used to compare with normal profiles to detect size and type of tumor.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>d=0</th>
<th>d=0.25</th>
<th>d=0.5</th>
<th>d=0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Θ=0</td>
<td>Θ=π/2</td>
<td>Θ=π</td>
<td>Θ=0</td>
</tr>
<tr>
<td>T1</td>
<td>37</td>
<td>36</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>T2</td>
<td>36.3</td>
<td>35.2</td>
<td>36.3</td>
<td>36.48</td>
</tr>
<tr>
<td>T3</td>
<td>35.3</td>
<td>34.09</td>
<td>35.29</td>
<td>35.4</td>
</tr>
<tr>
<td>T4</td>
<td>33.75</td>
<td>33.19</td>
<td>33.75</td>
<td>34.06</td>
</tr>
</tbody>
</table>

**Table 9:** Nodal temperatures for Ta=15, E=0, Case-II, Stage V and different eccentricities.

**Conclusions**

A two dimensional finite element model is proposed and employed to study the thermal patterns in the peripheral regions of human breast for a steady state case. The coaxial elliptical sectoral elements are employed in finite element discretization of the region to effectively incorporate the variation in parameters due to non homogeneity of skin and subdermal tissues of human breast. The model gives the result with high accuracy of 99.89% and these results are in agreement with the biological facts. However no theoretical or experimental results on thermal patterns in elliptical shaped human organs are available for comparison. The coaxial elliptical sectoral elements based finite element approach has been effectively used to model the realistic semi elliptical shaped of human breast varying in shape and size in terms of eccentricity, radius of minor and major axes due to different stages of breast developments. The present model is the most realistic model in terms of the shape and size in comparison to the models reported in the literature for the study of thermal patterns in human breast. On the basis of results it is concluded that the surface area of breast exposed to the environment changes with the changes in eccentricity due to different stages of development and it causes significant change in the thermal patterns of human breast. This thermal information of human breast due to change in shape and size of breast is of crucial importance in thermography for distinguishing the change in thermograms due to shape and size of breast and tumours. Further it can be concluded that the atmospheric temperature and rate of evaporation have significant effect on thermal patterns in human breast and this effect changes significantly with the change in shape and size of the breast. Finally the different stages of development of breast exhibit different thermal behaviours. The thermal information generated from such models can be useful for developing the strategy for thermography in relevance to atmospheric temperature, evaporation rates, shape and size of human breast for detection of breast cancer.

**References**


